

Phase extraction from optical interferograms in presence of intensity nonlinearity and arbitrary phase shifts

Thang Hoang,¹ Zhaoyang Wang,^{1,a)} Minh Vo,¹ Jun Ma,¹ Long Luu,¹ and Bing Pan²

¹Department of Mechanical Engineering, The Catholic University of America, Washington, DC 20064, USA

²Institute of Solid Mechanics, Beijing University of Aeronautics and Astronautics, Beijing 100191, China

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We present an advanced technique to retrieve phase from multiple optical interferograms containing intensity nonlinearity and random phase shifts, which are common in practice. The proposed algorithm employs a least-squares iteration scheme to detect harmonics up to the p th order and arbitrary phase shifts simultaneously, and the phase distribution can be accurately extracted from $(2p + 1)$ interferograms. The technique is validated by both computer simulation and real experimental results. © 2011 American Institute of Physics. [doi:10.1063/1.3614447]

Phase-shifting technique is commonly employed in optical interferometry because of its automatic, easy, accurate, and full-field characteristics.¹ The technique has been utilized extensively in various applications, such as three-dimensional holography²⁻⁴ and optical imaging microscopy.⁵⁻⁷ The most widely used conventional phase-shifting technique is based on the assumptions that the captured signal rigorously follows a simple sinusoidal function and the phase shifts are exactly the ideal values. In practice, however, none of the assumptions holds true. The two primary sources of systematic errors in extracting phase distributions are systematic phase-shift error, which is due to phase-shift miscalibration or nonlinear response of the phase shifter, and nonsinusoidal waveform of the signal, which is due to nonlinearity of the detector or multiple-beam interference.^{8,9} In addition, random phase-shift error is also very common in real applications due to the imperfect mechanism of phase-shifting and other practical issues such as vibration. To compensate these errors, numerous algorithms have been developed. These algorithms can cope with either the phase-shift error¹⁰⁻¹⁸ or the nonsinusoidal error,^{19,20} but not both. The reason is that the effects of the two errors on phase extraction are coupled, and they must be solved simultaneously.

In this Letter, a least-squares-based iterative algorithm to directly address the aforementioned issues is presented. The algorithm offers a way to compensate the effect of intensity nonlinearity and detect the actual phase shifts, which leads to an extraction of full-field phase map from multiple interferograms with high fidelity.

Mathematically, the existence of nonlinear response in optical interferometry brings high-order harmonics to the actual interferograms. Therefore, the intensity of a real interferogram can be theoretically expressed as

$$I_{ij}^t = \sum_{k=0}^p b_{ijk} \cos\{k[\phi_j + \delta_i]\}, \quad (1)$$

where i denotes the i th phase-shifted image ($i = 1, 2, \dots, M$), j denotes an arbitrary point in the image ($j = 1, 2, \dots, N$), b_{ij0} is the background intensity, b_{ijk} is the intensity modulation

amplitude of the k th order harmonics ($k \geq 1$), ϕ_j is the phase, δ_i is the i th phase-shift amount, and p is the highest significant harmonic order of the captured image. The equation used by the conventional phase-shifting technique is a special case of Eq. (1) with $p = 1$ and $\delta_i = (i - 1)2\pi/M$. In reality, as mentioned previously, this is generally not true.

The proposed technique is an iterative algorithm involving three steps in each iteration cycle.

Step 1: Determination of arbitrary phase shifts. In this step, the coefficient b_{ijk} is assumed to be a constant for each frame but can vary from a frame to another one, i.e., $b_{ijk} = B_{ik}$. Under this assumption, two sub-steps are applied to find the phase shifts. The first one is to calculate B_{i0}, \dots, B_{ip} with known phase information (i.e., ϕ_j and δ_i obtained from previous iteration cycle or from an existing phase-shifting algorithm (Ref. 21) for initial estimation). A least-square error accumulated from all the pixels in the i th image can be written as

$$S_i = \sum_{j=1}^N \left\{ \sum_{k=0}^p B_{ik} \cos[k(\phi_j + \delta_i)] - I_{ij} \right\}^2, \quad (2)$$

where I_{ij} denotes the captured interferogram intensity.

Minimizing S_i with respect to B_{ik} yields

$$A^{(i)} X^{(i)} = Y^{(i)}, \quad (3)$$

where $A^{(i)}$ is a $(p + 1) \times (p + 1)$ matrix, and $X^{(i)}$ and $Y^{(i)}$ are $(p + 1) \times 1$ column vectors. Their elements are

$$A_{kl}^{(i)} = \sum_{j=1}^N \cos[k(\phi_j + \delta_i)] \cos[l(\phi_j + \delta_i)],$$

$$X_k^{(i)} = B_{ik}, \quad (4)$$

$$Y_k^{(i)} = \sum_{j=1}^N \cos[k(\phi_j + \delta_i)] I_{ij},$$

where $k, l = 0, 1, \dots, p$. By solving for $X^{(i)}$ in Eq. (3), B_{i0}, \dots, B_{ip} can be obtained. Next, in the second sub-step, these new background and modulation amplitude values are combined with the phase distribution ϕ_j to update the phase-shift amount δ_i . This can be done through searching for the

^{a)}Electronic mail: wangz@cua.edu.

new phase shift in a range around the current one to minimize the following least-squares sum

$$S_i = \sum_{j=1}^N \left\{ \sum_{k=0}^p B_{ik} \cos[k(\phi_j + \delta_i + \Delta_i)] - I_{ij} \right\}^2, \quad (5)$$

where $\Delta_i \in (-\Delta_{max}, \Delta_{max})$ and Δ_{max} is a predefined threshold value. In this Letter, Δ_{max} is set to $\pi/2$ or 90° , and the finest searching step increment is 0.0001 rad or 0.0057° . After this step, the phase shift is updated as an adjustment of the previous value by $\delta_i^{(new)} = \delta_i^{(old)} + \Delta_i$.

Step 2: Determination of phase distribution. In this step, it is assumed that the coefficient b_{ijk} is a function of pixel location and does not have a frame-to-frame variation, i.e., $b_{ijk} = B_{jk}$. Similar to step 1, two sub-steps are conducted here. With phase ϕ_j obtained from the previous cycle and δ_i updated in step 1 of the current cycle, the least-squares error is

$$S_j = \sum_{i=1}^M \left\{ \sum_{k=0}^p B_{jk} \cos[k(\phi_j + \delta_i)] - I_{ij} \right\}^2, \quad (6)$$

and this yields

$$A^{(j)} X^{(j)} = Y^{(j)}, \quad (7)$$

where the elements of matrix $A^{(j)}$ and column vectors $X^{(j)}$ and $Y^{(j)}$ are

$$\begin{aligned} A_{kl}^{(j)} &= \sum_{i=1}^M \cos[k(\phi_j + \delta_i)] \cos[l(\phi_j + \delta_i)], \\ X_k^{(j)} &= B_{jk}, \\ Y_k^{(j)} &= \sum_{i=1}^M \cos[k(\phi_j + \delta_i)] I_{ij}. \end{aligned} \quad (8)$$

Solving Eq. (7) yields B_{j0}, \dots, B_{jp} . Then, similarly to step 1, the phase ϕ_j can be updated by searching for Δ_j in a range of $(-\Delta_{max}, \Delta_{max})$ around its current value to minimize the least-squares sum

$$S_j = \sum_{i=1}^M \left\{ \sum_{k=0}^p B_{jk} \cos[k(\phi_j + \Delta_j + \delta_i)] - I_{ij} \right\}^2. \quad (9)$$

In this way, the phase is updated as $(\phi_j + \Delta_j)$.

Step 3: Convergence check. The iteration continues until the following convergence criterion is satisfied for each phase-shift value δ_i :

$$|(\delta_i^{(n)} - \delta_1^{(n)}) - (\delta_i^{(n-1)} - \delta_1^{(n-1)})| < \epsilon, \quad (10)$$

where n denotes the number of iteration cycles and ϵ is a predefined accuracy threshold (e.g., 10^{-4}).

Equation (1) involves unknowns b_{ijk} , ϕ_j , and δ_i . Because the actual b_{ijk} generally does not have a frame-to-frame variation, the total number of unknowns is $[(p+2)N + M]$. Considering that the number of available equations is (MN) , at least $M = (p+2)N/(N-1)$ phase-shifted interferograms are required for the phase extraction. In reality, because the

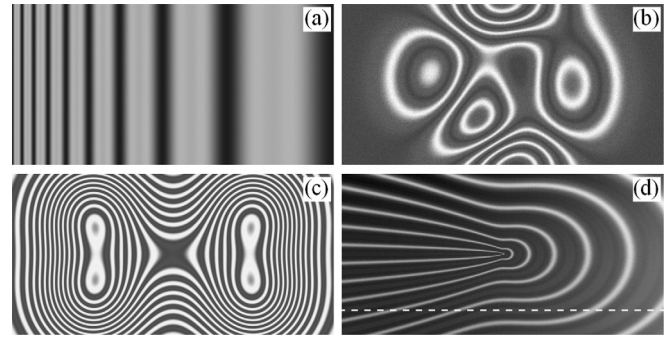


FIG. 1. Interferograms used in the simulation.

proposed algorithm employs a least-squares approach to solve for the coupled unknowns, more images are typically demanded to ensure a trustworthy processing. We have conducted numerous simulation tests to find the proper M for each different p . The result shows that using four images for $p=1$ or $(2p+1)$ images for $p>1$ is capable of providing reliable analysis for all the tests, although some only require as few as $(p+3)$ images. The tests also verify that the selected p can be larger than the actual highest harmonics order and more than $(2p+1)$ interferograms may be used for a selected p . If two or more interferograms have very close phase shifts that make the pixel-by-pixel difference between these interferograms indistinguishable, then more than $(2p+1)$ interferograms may be required to ensure a correct convergence and reliable analysis.

Figure 1 shows four representative interferograms used in the simulation, where case *a* is noise-free, case *b* involves high noise levels, case *c* contains high-frequency fringes, and case *d* includes nonuniform background, and the p values are 2, 3, 4, and 5, respectively. Table I illustrates a comparison of the root-mean-square errors (RMSE) of the full-field phase distributions extracted using the conventional algorithm, Hibino's algorithm,¹⁰ advanced iteration algorithm (AIA), and the proposed algorithm. Hibino's algorithm and AIA are selected because the former is insensitive to quadratic and nonuniform phase shifts and the latter is among the best algorithms for analyzing arbitrarily phase-shifted interferograms in practice. The results clearly demonstrate the validity of the proposed algorithm. It is noteworthy that other simulation tests show that both the AIA and the proposed algorithms provide accurate results when there is no intensity nonlinearity (i.e., $p=1$).

To avoid redundancy, only the simulation case *d* is elaborated here to demonstrate the effectiveness of the proposed algorithm. In the simulation, arbitrarily phase-shifted moiré interferograms representing the deformation field around a

TABLE I. RMSE of the phase distributions extracted by various algorithms (unit: radian).

	Case <i>a</i>	Case <i>b</i>	Case <i>c</i>	Case <i>d</i>
Conventional	0.3854	0.5033	0.2779	0.2004
Hibino	0.4217	0.4357	0.2159	0.3572
AIA	0.2701	0.1736	0.1245	0.1004
Proposed	0.0101	0.0370	0.0127	0.0236

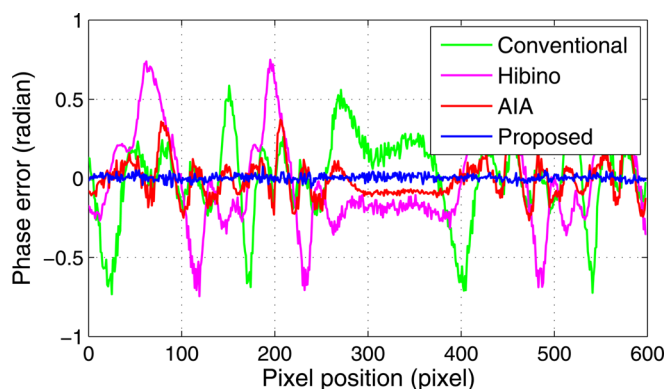


FIG. 2. (Color online) Errors of extracted phase distributions along a line.

crack are generated. The simulation parameters are $p = 5$, $b_{0-5} = \{60 + 68 * (i/w)^2, 30 + 40 * (j/h)^2, 30, 15, 8, 4\}$ and $\delta_{1-11} = \{0, 0.3491, 2.0944, 5.2360, 4.3633, 3.3161, 1.5708, 3.6652, 1.3963, 4.8869, 2.6180\}$. Here, (i, j) indicates pixel location and w and h denote the image width and height, respectively. In addition, Gaussian noise with mean $\mu = 0$ and standard deviation $\sigma = 5$ is added to the interferograms. Figure 2 shows the phase distributions along a line highlighted in the original interferogram, Fig. 1(d). It is evident that the proposed algorithm provides substantially better results than the existing algorithms due to its capability of handling high-order harmonics effect.

The proposed algorithm has also been applied to analyze randomly phase-shifted interferograms acquired in the mechanics measurement of a beam subjected to bending load. Since p cannot be determined prior to the interferogram analysis, a relatively large number of images (15 images to allow the maximum p to be 7) are captured in the experiment. The analysis with the proposed algorithm shows that stable results can be obtained if p is set to 4 or larger, and this reveals that the highest harmonics order in the interferograms is $p = 4$. Figure 3 shows one of the captured interferograms as well as the phase maps extracted by the existing AIA and the proposed algorithms. The experiment verifies the validity and feasibility of the proposed algorithm.

In summary, we have presented an iterative algorithm to extract phase information from optical interferograms involving both intensity nonlinearity and arbitrary phase shifts, which are the two errors typically encountered in real applications. The algorithm is applicable of eliminating the

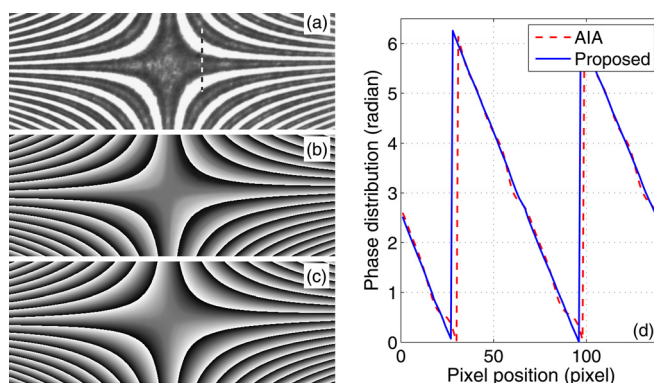


FIG. 3. (Color online) Analysis results of real interferograms. (a) A captured interferogram, phase maps extracted by (b) the existing AIA and (c) the proposed algorithms, and (d) phase distribution along a line highlighted in the initial image.

effect of arbitrary phase shifts and harmonics up to the p th order with $(2p + 1)$ or more interferograms.

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